Chin. Quart. J. of Math.
2016, 31 (3): 279—287

Periodic Solutions of a Cooperative System with State Feedback Control

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Abstract: In this paper, we propose a semi-continuous dynamical system to study the cooperative system with feedback control. Based on geometrical analysis and the analogue of Poincaré criterion, the existence and stability of the positive order one periodic solutions are given. Numerical results are carried out to illustrate the feasibility of our main results.

Key words: cooperative system; feedback control; order one periodic solution; orbitally asymptotically stable

2000 MR Subject Classification: 92B05, 34A37, 34D05

CLC number: O29 **Document code**: A

Article ID: 1002–0462 (2016) 03–0279–09

DOI:10.13371/j.cnki.chin.q.j.m.2016.03.006

§1. Introduction

As one of the three important ecological systems, cooperative system has attracted lots of researchers' attention. However, compared with predator-prey system and competitive system, still less work has been done. Cooperative interactions are reciprocally beneficial relationships between organisms. In virtue of their widespread occurrence in ecological system and human society, a large number of ODE and DDE models have been proposed to study it[1-4]. B S Goh in[5] gave a simple test for global stability in a large class of nonlinear models of mutualism. X Z

Received date: 2016-02-10

Foundation item: Supported by the National Natural Science Foundation of China(11671346, 11501489, 11371306, 11301453); Supported by the Department of Education of Henan Province(14B110034); Supported by the Nanhu Scholars Program of XYNU, Foundation and Frontier Project of Henan(152300410019); Supported by the Youth Teacher Foundation of XYNU(2016GGJJ-14)

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He and K Gopalsamy studied the persistence, attractively and delay in Facultative Mutualism in[6]. Time delays in cooperative system were also discussed[7-9].

With the development of the impulsive differential equation theory in recent decades, its application in population dynamics is also becoming common. A lot of discontinuous, impulsive phenomena in the natural ecological systems are studied by constructing impulsive differential equation models. One of such phenomena getting increasingly attention is human intervention in the optimal management of renewable resources(for example, exploitation of biological resources and the harvest of species in fishery, forestry and wildlife management)[10-17]. A typical characteristic of this kind of intervention is that the behavior of harvest is relatively instantaneous, so impulsive dynamical systems can describe it much better. However, such research on species with cooperative interaction is rarely seen in the literature. Z Yao[18] studied a cooperative system with impulsive harvest, but the harvest was operated on fixed time without knowing the biomass of the species. A highly possible risk of this kind of harvest is excessive exploitation, even resource exhaustion. To improve the harvesting styles, we introduce a reliable real time monitoring system to estimate the biomass of the species. Only when the biomass of the species according to the monitoring system reaches a certain level, the behavior of harvest can be carried out, otherwise, no harvest is permitted. Motivated by the above facts, in this paper, we investigate the cooperative system with state feedback impulsive control of renewable resource.

Let $x(t)$ and $y(t)$ denote population densities of the two cooperative species at time t, respectively. Suppose both populations grow logistically with carrying capacities given by r_1/a and r_2/d, intrinsic growth rates governed by r_1 and r_2 and the two species interaction coefficient governed by b and c. The impulsive harvest is modeled by the following semi-continuous system

$$\begin{cases} \left.\begin{aligned} \frac{\mathrm{d}x(t)}{\mathrm{d}t} &= x(t)(r_1 - ax(t) + by(t)), \\ \frac{\mathrm{d}y(t)}{\mathrm{d}t} &= y(t)(r_2 + cx(t) - dy(t)), \end{aligned}\right\} \quad y < h, \\ \left.\Delta x(t) = 0, \quad \Delta y(t) = -\beta y(t),\right\} \quad y = h, \\ x(0) = x_0, \quad y(0) = y_0, \end{cases} \tag{1.1}$$

where $x_0 \geq 0$ and $y_0 \geq 0$ denote initial densities of the two cooperative species, respectively. h is an adjustable constant threshold value for the density of the second species-when the population density y reaches the threshold value, the impulsive harvest of this species with proportion p is performed. Throughout this paper we assume that the initial density of the y-population is always less than h. Otherwise, the initial values are taken after an impulsive application.

The paper is organized as follows. In Section 2, some notations and preliminary results are provided. In Section 3, we mainly discuss the existence and orbitally stability of the periodic solution of system (1.1). The paper ends with some numerical simulations and a brief discussion.

§2. Preliminaries

Without impulsive effect, system (1.1) can be written as

$$\begin{cases} \dfrac{\mathrm{d}x(t)}{\mathrm{d}t} = x(t)(r_1 - ax(t) + by(t)), \\ \dfrac{\mathrm{d}y(t)}{\mathrm{d}t} = y(t)(r_2 + cx(t) - dy(t)). \end{cases} \tag{2.1}$$

For system (2.1), there are always three boundary equilibria: an unstable node $O(0,0)$ and two saddle points $A(r_1/a, 0)$ and $B(0, r_2/d)$. If $ad - bc > 0$, there is another interior node (x^*, y^*) that is globally stable in the first quadrant, where $x^* = \frac{r_1 d + r_2 b}{ad-bc}, y^* = \frac{ar_2 + cr_1}{ad-bc}$.

Throughout this paper, we assume $h < \frac{ar_2+cr_1}{ad-bc}$ when $ad-bc > 0$. In practice, if $h > \frac{ar_2+cr_1}{ad-bc}$, the population level of y will not be in a high state because it will tend to $\frac{ar_2+cr_1}{ad-bc}$ eventually without human intervention.

To discuss the dynamics of system (1.1), we firstly denote two cross-sections in the vector field of system (1.1) by

$$\sum^{P} = \{(x,y)|x>0, y=(1-p)h\}, \qquad \sum^{I} = \{(x,y)|x>0, y=h\}.$$

In the following, we construct a Poincaré map in the way[19] and give the definition of periodic solutions.

Suppose that the trajectory of system (1.1) starts from the point $P_n(x_n, (1-p)h)$ on $\sum^P$ firstly intersects $\sum^I$ at point $P'_n(x'_n, h)$, after impulsive effect it jumps to $\sum^P$ at point $P_{n+1}(x_{n+1}, (1-p)h)$. Then the associated Poincaré map defined on $\sum^P$ is given by

$$F: \sum^{P} \to \sum^{P}, \quad (x_n, (1-p)h) \to (x_{n+1}, (1-p)h).$$

Let $(x(t),\ y(t))$ be a solution of system (1.1) which starts from point $P_0(x_0, (1-p)h)$ on $\sum^P$. The trajectory with the initial condition P_0 firstly intersects the cross-section Σ^I at point $P'_0(x'_0, h)$, then jumps to the point $P_1(x_1, (1-p)h)$ on the cross-section $\sum^P$. Repeating the above process, we can get the impulsive sequence: $\{P_k(x_k, (1-p)h)\}(k \in N)$.

Definition 2.1[20] A solution $(x(t), y(t))$ of system (1.1) through the point $(x_0, (1-p)h)$ at $t=0$ is said to be order n periodic if $x_0 = x_n$.

Remark 1 If $F(x_0, (1-p)h) = (x_0, (1-p)h), (x_0, (1-p)h) \in \sum^P$, then system (1.1) has an order one periodic solution passing through point $(x_0, (1-p)h)$.

Remark 2 The Poincaré map $F(x, (1-p)h)$ is continuous in x.

For the stability of the order n periodic solution, consider the following autonomous system with impulsive effects

$$\begin{cases} \dfrac{\mathrm{d}x}{\mathrm{d}t} = P(x,y), \quad \dfrac{\mathrm{d}y}{\mathrm{d}t} = Q(x,y), \quad \varphi(x,y) \neq 0, \\ \triangle x = \alpha(x,y), \quad \triangle y = \beta(x,y), \quad \varphi(x,y) \neq 0, \end{cases} \tag{2.2}$$

where $P(x,y)$ and $Q(x,y)$ are continuous differential functions and $\varphi(x,y)$ is a sufficiently smooth function with $\nabla\varphi(x,y) \neq 0$. Let $(\xi(t),\eta(t))$ be a positive T-periodic solution of system (2.2). By results in Simeonov and Bainov[21], we have the following Lemma.

Lemma 2.1(Analogue of Poincaré Criterion) If the Floquet multiplier μ satisfies the condition $\mu < 1$, where

$$\mu = \prod_{k=1}^{n} \Delta_k \exp\Big[\int_0^T \Big(\frac{\partial P}{\partial x}(\xi(t),\eta(t)) + \frac{\partial Q}{\partial y}(\xi(t),\eta(t))\mathrm{d}t\Big)\Big]$$

with

$$\Delta_k = \frac{P_+\Big(\dfrac{\partial\beta}{\partial y}\dfrac{\partial\varphi}{\partial x} - \dfrac{\partial\beta}{\partial x}\dfrac{\partial\varphi}{\partial y} + \dfrac{\partial\varphi}{\partial x}\Big) + Q_+\Big(\dfrac{\partial\alpha}{\partial x}\dfrac{\partial\varphi}{\partial y} - \dfrac{\partial\alpha}{\partial y}\dfrac{\partial\varphi}{\partial x} + \dfrac{\partial\varphi}{\partial y}\Big)}{P\dfrac{\partial\varphi}{\partial x} + Q\dfrac{\partial\varphi}{\partial y}},$$

where P, Q, $\frac{\partial\alpha}{\partial x}$, $\frac{\partial\alpha}{\partial y}$, $\frac{\partial\beta}{\partial x}$, $\frac{\partial\beta}{\partial y}$, $\frac{\partial\varphi}{\partial x}$ and $\frac{\partial\varphi}{\partial y}$ are calculated at the point $(\xi(\tau_k),\eta(\tau_k))$, $P_+ = P(\xi(\tau_k^+),\ \eta(\tau_k^+))$, $Q_+ = Q(\xi(\tau_k^+),\eta(\tau_k^+))$ and τ_k is the time of the kth jump. Then, $(\xi(t),\eta(t))$ is orbitally asymptotically stable.

§3. Existence and Stability of Periodic Solutions

In this section, we mainly discuss the existence and stability of the order one periodic solution of the system (1.1) by geometrical analysis and the analogue of Poincaré criterion.

To illustrate the existence of the positive periodic solution, we will turn to the geometrical construction of the phase space of system (1.1).

Theorem 3.1 Assume that $ad - bc \leq 0$(or $ad - bc > 0, h < \frac{ar_2+cr_1}{ad-bc}$), then system (1.1) has a positive order one periodic solution.

Proof Suppose that the isocline $r_1 - ax + by = 0$ intersects the horizontal lines $y = (1-p)h$ and $y = h$ at points $C(x_C,(1-p)h)$ and $D(x_D,h)$, respectively. Points $E(x_E,(1-p)h)$ and $F(x_F,(1-p)h)$ are on the cross-section Σ^P, where $x_E = x_D$ and point F is on the right and next to the point C(see Fig. 1).

Because $r_1 - ax + by = 0$ is the vertical isocline, variable x increases above this vertical isocline in the vector field and decreases in the lower half of the vector field. Consider the following two special trajectories of system (1.1).

(Γ_1) The trajectory starting from point E must intersect the cross-section Σ^P at a point $E^{'}(x_{E'},h)$, where $x_{E'} < x_D$. The point $E^{'}$ is mapped to the cross-section Σ^I at point $E_1(x_{E_1},(1-p)h)$ after impulsive effect, where $x_{E_1} < x_E = x_D$(because $x_{E_1} = x_{E'}$).

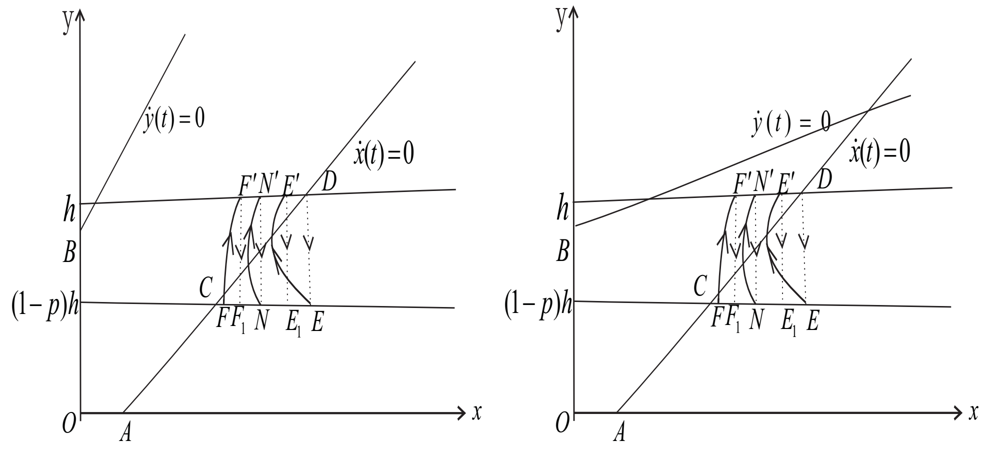

Fig. 1 Existence of order one periodic solution of system (1.1) when $ad-bc\le 0$ and when $ad-bc>0$ and $h<\frac{ar_2+cr_1}{ad-bc}$.

(Γ_2) The trajectory starting from point F must intersect the cross-section Σ^P at a point $F'(x_{F'},h)$, where $x_{F'}<x_{E'}$(because distinct trajectories do not intersect and point F is next to point C). The point F' is mapped to the cross-section Σ^I at point $F_1(x_{F_1},(1-p)h)$ after impulsive effect, where $x_F<x_{F_1}<x_{E_1}$.

For the points E and F on the cross-section Σ^P, we have a conclusion about the Poincaré map $F(E)=E_1, F(F)=F_1$. That is to say

$$F(x_E,(1-p)h)=(x_{E_1},(1-p)h),\ F(x_F,(1-p)h)=(x_{F_1},(1-p)h).$$

The Poincar$\acute{e}$ map $F(x,(1-p)h)$ is continuous in x, so there must exist a point N between the points E and F such that $F(N)=N$ due to $x_{E_1}<x_E$ and $x_{F_1}>x_F$. According to Definition 2.1, the system (1.1) has an order one periodic solution that passes through point N.

Theorem 3.2 If $ad-bc\le 0$(or $ad-bc>0, h<\frac{ar_2+cr_1}{ad-bc}$), $r_2-r_1-2(b+d)(1-p)h+bh+\frac{c(r_1+bh)}{a}<0$ and $r_2+\frac{c(r_1+b(1-p)h)}{a}-2dh>0$, then the order one periodic solution is orbitally asymptotically stable.

Proof Suppose the intersection of the order one periodic solution and the cross-section Σ^P is $N(x_N,(1-p)h)$ and the period of the periodic solution is T, then we have $x_C=\frac{r_1+b(1-p)h}{a}<x_N<x_E=\frac{r_1+bh}{a}$. Denote this periodic solution by $(\xi(t),\eta(t))$, we can easily get $(\xi(T),\eta(T))=(x_N,h)$ and $(\xi(T^+),\eta(T^+))=(x_N,(1-p)h)$.

According to Lemma 2.1, we have $P(x,y)=x(r_1-ax+by), Q(x,y)=y(r_2+cx-dy), \varphi(x,y)=y-h, \alpha(x,y)=0$ and $\beta(x,y)=-py$. An easy calculation shows that

$$\frac{\partial P}{\partial x}=r_1-2ax+by,\ \frac{\partial Q}{\partial y}=r_2+cx-2dy,$$

$$\frac{\partial\alpha}{\partial x}=\frac{\partial\alpha}{\partial y}=\frac{\partial\beta}{\partial x}=\frac{\partial\varphi}{\partial x}=0,\ \frac{\partial\beta}{\partial y}=-p, \frac{\partial\varphi}{\partial y}=1.$$

Obviously, the periodic solution satisfies $x_C = \frac{r_1+b(1-p)h}{a} < \xi(t) < x_E = \frac{r_1+bh}{a}$, $(1-p)h \leq \eta(t) \leq h$, so we have

$$\begin{aligned}
&\int_0^T (\frac{\partial P}{\partial x}(\xi(t),\eta(t)) + \frac{\partial Q}{\partial y}(\xi(t),\eta(t))\mathrm{d}t \\
=& \int_0^T (r_1 + r_2 - 2a\xi(t) + c\xi(t) + b\eta(t) - 2d\eta(t))\mathrm{d}t \\
\leq& \int_0^T (r_1 + r_2 - 2a\frac{r_1 + b(1-p)h}{a} + c\frac{r_1 + bh}{a} + bh - 2d(1-p)h)\mathrm{d}t \\
=& \int_0^T (r_2 - r_1 - 2(b+d)(1-p)h + bh + \frac{c(r_1+bh)}{a})\mathrm{d}t. \qquad (3.1)
\end{aligned}$$

If $r_2 - r_1 - 2(b+d)(1-p)h + bh + \frac{c(r_1+bh)}{a} < 0$ holds, we can easily obtain

$$\exp\,[\int_0^T (\frac{\partial P}{\partial x}(\xi(t),\eta(t)) + \frac{\partial Q}{\partial y}(\xi(t),\eta(t))\mathrm{d}t)] < 1. \qquad (3.2)$$

Consider the function $f(y) = y(r_2 + cx_N - dy)$ on $[(1-p)h, h]$. Obviously, for $(1-p)h < y < h$, we have $f(y) > 0$ and

$$f'(y) = r_2 + cx_N - 2dy \geq r_2 + \frac{c[r_1 + b(1-p)h]}{a} - 2\mathrm{d}h.$$

If $r_2 + \frac{c[r_1+b(1-p)h]}{a} - 2dh > 0$, then $f'(y) > 0$ for $(1-p)h < y < h$ and $0 < f((1-p)h) < f(h)$.

According to Lemma 2.1, if $r_2 + \frac{c[r_1+b(1-p)h]}{a} - 2\mathrm{d}h > 0$, we get

$$0 < \Delta_k = \frac{(1-p)h(r_2 + cx_N - d(1-p)h)}{h(r_2 + cx_N - \mathrm{d}h)} = \frac{f((1-p)h)}{f(h)} < 1.$$

To sum up, if $r_2 - r_1 - 2(b+d)(1-p)h + bh + \frac{c(r_1+bh)}{a} < 0$ and $r_2 + \frac{c(r_1+b(1-p)h)}{a} - 2\mathrm{d}h > 0$, then

$$\mu = \Delta_k \exp\,[\int_0^T (\frac{\partial P}{\partial x}(\xi(t),\eta(t)) + \frac{\partial Q}{\partial y}(\xi(t),\eta(t))\mathrm{d}t)] < 1.$$

Due to Lemma 2.1, the positive periodic solution is orbitally asymptotically stable and has the asymptotic phase property. That completes the proof.

§4. Numerical Simulations and Discussions

In this section, we shall use numerical simulations to illustrate the feasibility of our main results. In Fig. 2 and Fig. 3, we let $r_1 = 1,\ r_2 = 1,\ a = 0.5,\ b = 0.5,\ c = 0.1,\ d = 0.1,\ p = 0.3$ and $h = 1$. Then $ad - bc = 0$, and we can see that without impulsive effect, the system (2.1) does not has a positive equilibrium and the densities of the two species quickly grow up (see Fig. 2). By simple calculation, we obtain $r_2 - r_1 - 2(b+d)(1-p)h + bh + \frac{c(r_1+bh)}{a} = -0.04 < 0$ and $r_2 + \frac{c(r_1+b(1-p)h)}{a} - 2dh = 3.5 > 0$. According to Theorem 3.2, the system (1.1) has an order one periodic solution that is orbitally asymptotically stable. Fig. 3 reveals that the state-dependent impulsive harvests of the second species maintain the system at a sustained oscillatory state. It also exhibits the orbital stability of the order one periodic solution.

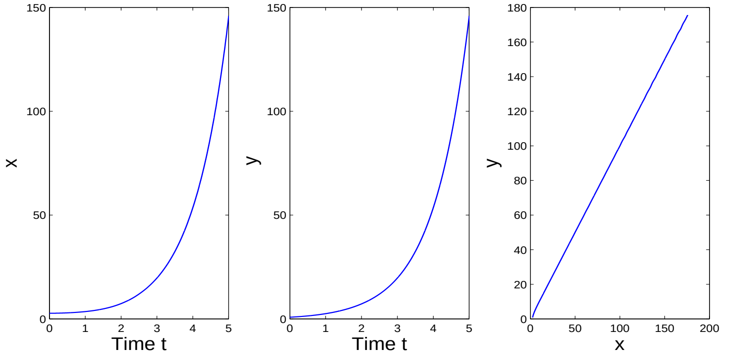

Fig. 2 The time series and the fortrait phase of the system (2.1) when $ad - bc \leq 0$ with initial values $(2, 0.8)$.

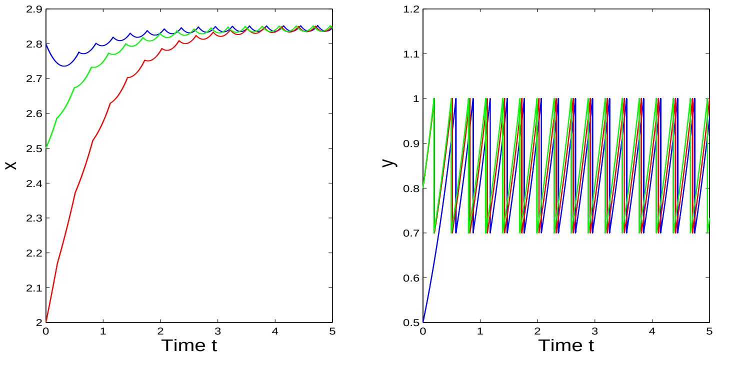

Fig. 3 Existence and orbitally asymptotically stable of the order one periodic solutions of the system (1.1) with different initial values $(2, 0.8), (2.5, 0.8)$ and $(2.8, 0.5)$ when $ad - bc < 0$.

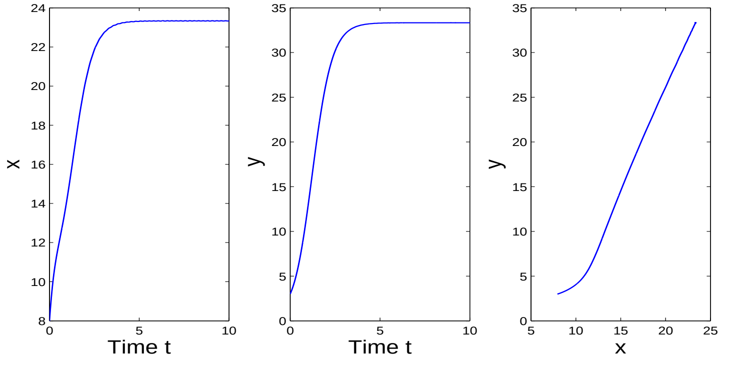

Fig. 4 The time series and the portrait phase of the system (2.1) when $ad - bc > 0$ with initial values $(8, 3)$.

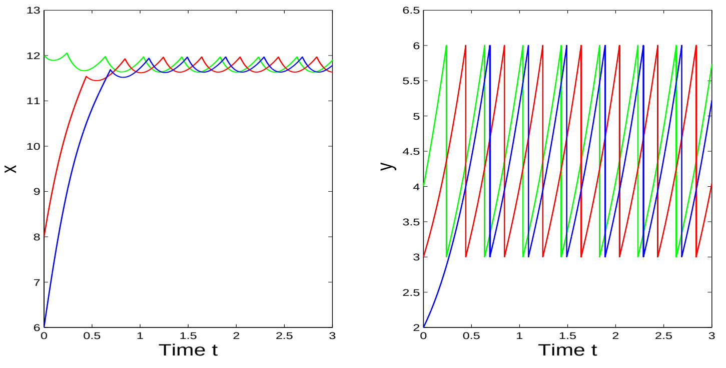

Fig. 5 Existence and orbitally asymptotically stable of the order one periodic solutions of the system (1.1) with different initial values $(6,2)$, $(8,3)$ and $(12,4)$ when $ad-bc>0$.

In Fig. 4 and Fig. 5, we let $r_1=5,\ r_2=1,\ a=0.5,\ b=0.2,\ c=0.1,\ d=0.1,\ p=0.5$ and $h=6$. Then $ad-bc=0.03>0$ and $\frac{ar_2+cr_1}{ad-bc}=33.3>h$, and we can see that without impulsive effect, the system (2.1) does has a positive equilibrium and the densities of the two species tends to a high level(see Fig. 4). By simple calculation, we obtain $r_2-r_1-2(b+d)(1-p)h+bh+\frac{c(r_1+bh)}{a}=-3.36<0$ and $r_2+\frac{c(r_1+b(1-p)h)}{a}-2dh=0.92>0$. According to Theorem 3.2, the system (1.1) also has an order one periodic solution that is orbitally asymptotically stable. From Fig. 5, we can see that, controlled by a predefined threshold value h, both densities of the two cooperative species are controlled in a relatively low level and the state-dependent impulsive harvests also maintain the system at a sustained stable oscillatory state.

We build a cooperative system with state feedback impulsive harvest in this paper. We assume the second species is a renewable resource that has high commercial value. To avoid over-exploitation of it, we introduce a real time monitoring system for the species' density, harvest can be carried out only when the species density reaches a adjustable predefined value. theorems 3.1 and 3.2 ensure that the cooperative system has a positive periodic solution under this state feedback control and the periodic solution is orbitally asymptotically stable provided some conditions are satisfied.

The existence and stability of the order one periodic solution ensure that the perturbation by the harvest in such an automated way can keep the species density under control. This illustrates that under reasonable control of the impulsive harvest yield, people can achieve economic benefits effectively and avoid excessive exploitation at the same time.

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